

FROM THE DE DONDER-WEYL HAMILTONIAN FORMALISM TO QUANTIZATION OF GRAVITY^a

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An approach to quantization of fields and gravity based on the De Donder-Weyl covariant Hamiltonian formalism is outlined. It leads to a hypercomplex extension of quantum mechanics in which the algebra of complex numbers is replaced by the space-time Clifford algebra and all space-time variables enter on equal footing. A covariant hypercomplex analogue of the Schrödinger equation is formulated. Elements of quantization of General Relativity within the present framework are sketched.

1 Introduction

A synthesis of Quantum Theory and General Relativity remains one of the fundamental problems in theoretical physics. Various approaches to the problem have been proposed which vary from ideas of the revision of our basic concepts about space-time or quantum theory to the latest developments in string (or M-) theory and quantum gravity. The problem of *quantization* of gravity represents only one of the attempts to reach the above mentioned synthesis^{1,2}. Again, there exist several approaches to quantization of gravity, of which the canonical quantization can be considered as the most straightforward one. Usually the canonical quantization is preceded by the Hamiltonian formulation. The Poisson(-Dirac) brackets found within the latter give rise to the commutation relations of quantum operators. The scheme can be applied, at least in principle, to General Relativity constituting the basis of the contemporary approaches to quantum gravity, such as the Wheeler-De Witt superspace formulation or the Ashtekar non-perturbative quantization program. However, the feature of the Hamiltonian formalism that a time dimension (or a foliation of space-like hypersurfaces) has to be specified prior to construction of the Hamiltonian formalism seems to be a rather unnatural restriction in the context of quantum theory of gravity. In fact, the latter is expected to imply intricate Planck-scale fluctuations of topology and even of a signature of the space-time which are hardly compatible with the possibility of specifying a natural time parameter. The widely discussed “Issue of Time” in quantum gravity and cosmology¹ may be viewed as another manifestation of conceptual

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problems which we face while applying traditional methods to the realm of quantum gravity. In view of the above, applicability of the standard Hamiltonian formalism to the problem of quantization of gravity can be questioned.

As an attempt to circumvent, or at least to shed new light on the above mentioned difficulties, we will discuss in what follows an approach to the quantization of fields and gravity which is based on a different generalization of the Hamiltonian formalism to field theory. This generalization, which is manifestly covariant and does not prerequire singling out of a time dimension, has been known in the calculus of variations as the De Donder-Weyl (DW) theory since the thirties, although its applications in physics have been rather rare^{3,4,5}.

The idea of the DW Hamiltonian formulation is that for a Lagrangian density $L = L(y^a, \partial_\mu y^a, x^\nu)$, where x^ν are space-time coordinates ($\mu, \nu = 1, \dots, n$), y^a are field variables ($a, b = 1, \dots, m$), and $\partial_\mu y^a$ are their space-time derivatives, we can define Hamiltonian-like variables $p_a^\mu := \partial L / \partial (\partial_\mu y^a)$ (polymomenta), and $H := \partial_\mu y^a p_a^\mu - L$ (DW Hamiltonian), such that the Euler-Lagrange field equations take the form of DW Hamiltonian equations

$$\partial_\mu y^a = \partial H / \partial p_a^\mu, \quad \partial_\mu p_a^\mu = -\partial H / \partial y^a. \quad (1)$$

These equations can be viewed as a field theoretic (multi-parameter, or “multi-time”) generalization of the Hamilton canonical equations. If so, it is quite natural to try to construct a quantization scheme in field theory which is related to this formulation, much like the standard canonical quantization is related to the Hamiltonian formalism in mechanics.

We proceed as follows. In Sec. 2 we outline elements of the quantization based on the Poisson brackets on differential forms which appear within the DW formulation^{5,6}. In particular, we put forward a covariant generalization of the Schrödinger equation to the present context. In Sec. 3 we extend this scheme to curved space-time. In Sec. 4 an application to the problem of quantization of General Relativity is sketched and a corresponding generalization of the Schrödinger equation is presented. Discussion is found in Sec. 5.

2 Elements of the quantization based on the DW theory

We have shown earlier that the DW Hamiltonian formulation has an analogue of the Poisson bracket operation defined on (horizontal) differential forms of various degrees p , $0 \leq p \leq n$ (n is the dimension of the underlying spacetime manifold), $F = \frac{1}{p!} F_{\mu_1 \dots \mu_p}(y^a, p_a^\mu, x^\nu) dx^{\mu_1} \wedge \dots \wedge dx^{\mu_p}$, which play the role of dynamical variables^{5,6}. This bracket possesses (graded) analogues of the algebraic properties of the usual Poisson bracket, the existence of which is crucial for quantization. More specifically, the bracket on differential forms in

DW theory leads to generalizations of a so-called Gerstenhaber algebra. This bracket also enables us to identify the canonically conjugate variables and to write the DW Hamiltonian field equations (1) in Poisson bracket formulation. The latter demonstrates that the bracket of a form with the DW Hamiltonian is related to the (total) exterior differentiation of the form. This observation will be important below for our conjecture about an analogue of the Schrödinger equation.

For our present purposes it is sufficient to use canonical brackets in the subalgebra of forms of degree 0 and $(n-1)$. Using the notation $\omega_\mu := (-1)^{(\mu-1)} dx^1 \wedge \dots \wedge \widehat{dx^\mu} \wedge \dots \wedge dx^n$ those are given by⁵

$$\{[p_a^\mu \omega_\mu, y^b]\} = \delta_a^b, \quad \{[p_a^\mu \omega_\mu, y^b \omega_\nu]\} = \delta_a^b \omega_\nu, \quad \{[p_a^\mu, y^b \omega_\nu]\} = \delta_a^b \delta_\nu^\mu, \quad (2a, b, c)$$

with other brackets vanishing. By quantizing according to the Dirac correspondence rule, we conclude from the commutator corresponding to (2a) that $\widehat{p_a^\mu \omega_\mu} = i\partial_a$ is the operator of partial differentiation with respect to the field variables. We argued^{7,8} that operators $\widehat{p_a^\mu}$ and $\widehat{\omega_\mu}$ can be represented by means of Clifford imaginary units (Dirac matrices) $\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = 2\eta_{\mu\nu}$,^b

$$\widehat{p_a^\mu} = -i\hbar\kappa\gamma^\mu\partial_a, \quad \widehat{\omega_\mu} = -1/\kappa\gamma_\mu \quad (3)$$

if the law of composition of operators when calculating the commutator corresponding to (2c) is assumed to be the symmetrized Clifford (=matrix) product. Note that the coefficient κ in (3) has the dimension of $length^{-(n-1)}$ and its value is expected to be “very large” in order to agree with the infinitesimal character of the $(n-1)$ -volume element ω_μ . A possible relation of κ to the ultra-violet cutoff scale is discussed in^{7,9}.

The realization of operators in terms of Clifford imaginary units implies a certain generalization of the formalism of quantum mechanics. Namely, whereas the conventional quantum mechanics is built up on complex numbers which are essentially the Clifford numbers corresponding to the *one*-dimensional space-time (= the time dimension in mechanics), the present approach to quantization of fields viewed as multi-parameter Hamiltonian systems (of the De Donder-Weyl type) makes use of the hypercomplex (Clifford) algebra of the corresponding space-time manifold^{10,11}.

This philosophy and other considerations based essentially on the correspondence principle enable us to conjecture the following generalization of the Schrödinger equation to the present framework

$$i\hbar\kappa\gamma^\nu\partial_\nu\Psi = \widehat{H}\Psi, \quad (4)$$

^bA slightly different representation⁷: $p_a^\mu = i\hbar\kappa\gamma^\mu\gamma\partial_a$ with $\gamma \sim \gamma_1\gamma_2\dots\gamma_n$, has been proven to be incompatible with the Ehrenfest principle, see Eq. (10) below.

in which the components of the wave function Ψ are functions of the space-time and field variables: $\Psi = \Psi(x^\mu, y^a)$. If Ψ is a Dirac spinor and \hat{H} is Hermitian with respect to the L^2 scalar product in y -space, Eq. (4) possesses a positive definite conserved quantity as a consequence of the conservation law

$$\frac{\delta}{\delta\sigma} \int_{\sigma} \omega_{\mu} \int dy \bar{\Psi} \gamma^{\mu} \Psi = 0, \quad (5)$$

where $\bar{\Psi}$ denotes the Dirac conjugate of Ψ , and σ is any space-like hypersurface. This makes possible a probabilistic interpretation of Ψ .

In the case of a system of scalar fields $\{y^a\}$ given by the Lagrangian

$$L = \frac{1}{2} \partial_{\mu} y^a \partial^{\mu} y_a - V(y) \quad (6)$$

the polymomenta and the classical DW Hamiltonian read

$$p_{\mu}^a = \partial_{\mu} y^a, \quad H = \frac{1}{2} p_{\mu}^a p_a^{\mu} + V(y). \quad (7)$$

The corresponding quantum counterpart of H is shown in ^{7,8} to be

$$\hat{H} = -\frac{1}{2} \hbar^2 \kappa^2 \partial_a \partial^a + V(y). \quad (8)$$

Note that for a single free scalar field $V(y) = (1/2\hbar^2)m^2 y^2$, so that \hat{H} is similar to the Hamiltonian operator of the harmonic oscillator in the space of field variables.

One of the reasons to believe that (4) is a reasonable candidate to a proper wave equation is that it is consistent with the Ehrenfest principle. By assuming^c

$$\langle \hat{O} \rangle := \int dy \bar{\Psi} \hat{O} \Psi \quad (9)$$

from equations (3), (4) and (8) we obtain⁸

$$\begin{aligned} \partial_{\mu} \langle \hat{p}_a^{\mu} \rangle &= -\partial_{\mu} \int dy \bar{\Psi} i \hbar \kappa \gamma^{\mu} \partial_a \Psi = \dots = -\langle \partial_a \hat{H} \rangle \\ \partial_{\mu} \langle \widehat{y_a \omega^{\mu}} \rangle &= -\kappa^{-1} \partial_{\mu} \int dy \bar{\Psi} \gamma^{\mu} y_a \Psi = \dots = \langle \widehat{p_a^{\mu} \omega_{\mu}} \rangle. \end{aligned} \quad (10)$$

^cIt should be noticed that the scalar product $\int dy \bar{\Psi} \Psi$ is not positive definite if Ψ is a Dirac spinor. However, another version⁹ of the Ehrenfest theorem can also be proven using the positive definite scalar product implied by (5).

By comparing the result with (1) we conclude that the classical field equations are fulfilled “in average” as a consequence of the representation of operators (3), the generalized Schrödinger equation (4), and formula (9) for the expectation values of operators. Note also that using an appropriate (hypercomplex) analogue of the quasiclassical ansatz for the wave function Ψ it is possible to derive^{7,8} from (4) and (8) the Hamilton-Jacobi equation corresponding to the DW canonical theory⁴.

3 Generalization to curved space-time

Our next step is to extend the generalized Schrödinger equation (4) to curved space-time given by a metric tensor $g_{\mu\nu}(x)$. To do this we have (i) to introduce x -dependent γ -matrices $\gamma^\mu(x)$ such that

$$\gamma_\mu(x)\gamma_\nu(x) + \gamma_\nu(x)\gamma_\mu(x) = 2g_{\mu\nu}(x),$$

(ii) to replace the partial space-time derivative in the left hand side of (4) with the covariant derivative ∇_μ , and (iii) to generalize the operator \hat{H} to a curved space-time background. Denoting $g := \det||g_{\mu\nu}||$ the resulting generalized Schrödinger equation in curved space-time assumes the form

$$i\hbar\kappa\sqrt{|g|}\gamma^\mu(x)\nabla_\mu\Psi = \hat{H}\Psi. \quad (11)$$

For the system of scalar fields $\{y^a\}$ on a curved background we obtain⁹

$$\hat{p}_a^\mu = -i\hbar\kappa\sqrt{|g|}\gamma^\mu\partial_a, \quad \hat{H} = -\frac{\hbar^2\kappa^2}{2}\sqrt{|g|}\frac{\partial^2}{\partial y^a\partial y_a} + \sqrt{|g|}V(y). \quad (12)$$

If the wave function Ψ in (11) is a Dirac spinor the covariant derivative $\nabla_\mu := \partial_\mu + \theta_\mu$ includes the spinor connection

$$\theta_\mu = \frac{1}{2}\theta_{AB\mu}\gamma^{AB}, \quad (13)$$

where $\gamma^{AB} := \frac{1}{2}(\gamma^A\gamma^B - \gamma^B\gamma^A)$, $\gamma^A\gamma^B + \gamma^B\gamma^A = 2\eta^{AB}$, and η^{AB} is the Minkowski metric. The coefficients of the spinor connection are given by

$$\theta_{B\mu}^A = -e_B^\nu\partial_\mu e_\nu^A + e_\alpha^A\Gamma_{\mu\nu}^\alpha e_B^\nu, \quad (14)$$

where the tetrad coefficients e_A^μ satisfy

$$g^{\mu\nu}(x) = e_A^\mu(x)e_B^\nu(x)\eta^{AB}, \quad (15)$$

as usual. Note that an analogue of the Ehrenfest theorem also holds here, at least for scalar fields, due to the well-known relation for the spinor connection: $\partial_\mu(\sqrt{|g|}e_A^\mu) = \sqrt{|g|}e_B^\mu\theta_{A\mu}^B$ (the proof is to be presented elsewhere⁹).

4 Application to General Relativity

Now we can sketch how the present framework can be applied to General Relativity. The wave function will depend on the metric (or tetrad) and the space-time variables, i.e. $\Psi = \Psi(x^\mu, g^{\alpha\beta})$ (or $\Psi = \Psi(x^\mu, e_A^\alpha)$). Here the metric components $g^{\alpha\beta}$ are not functions of space-time variables as in Sec. 3. They enter rather as fibre coordinates of the bundle of symmetric rank two tensors over the space-time, with classical metrics $g^{\alpha\beta}(x)$ being sections in this bundle. With $g^{\mu\nu} =: e_A^\mu e_B^\nu \eta^{AB}$ we introduce γ -matrices $\gamma^\mu := e_A^\mu \gamma^A$ which fulfill

$$\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = 2g_{\mu\nu}.$$

Now, equations (4) and (11) in the case of quantum General Relativity generalize to

$$i\hbar\kappa\sqrt{|g|}\gamma^\mu\widehat{\nabla}_\mu\Psi = \widehat{H}\Psi, \quad (16)$$

where \widehat{H} is the operator of the DW Hamiltonian density and $\widehat{\nabla}_\mu$ denotes the operator corresponding to the covariant derivative in which the connection coefficients are replaced by appropriate differential operators (cf. Eq. (20)). Note that in gravitation theory it seems very natural to relate the constant κ with a Planck scale quantity, so that we can expect $\kappa \sim l_{Pl}^{-(n-1)}$.

In order to construct operators entering Eq. (16) we need to represent General Relativity in a DW-like Hamiltonian form. This issue has already been considered by several authors^{3,12}. However, the formulation given by Hořava¹³ seems to be the most suitable for our purposes. Using the metric density $h^{\alpha\beta} := \sqrt{|g|}g^{\alpha\beta}$ as a field variable and by defining the quantity

$$8\pi G Q_{\beta\gamma}^\alpha := \frac{1}{2}[\delta_\beta^\alpha \Gamma_{\gamma\delta}^\delta + \delta_\gamma^\alpha \Gamma_{\beta\delta}^\delta] - \Gamma_{\beta\gamma}^\alpha, \quad (17)$$

and the DW Hamiltonian density

$$H(h^{\alpha\beta}, Q_{\beta\gamma}^\alpha) := 8\pi G h^{\alpha\gamma}[Q_{\alpha\beta}^\delta Q_{\gamma\delta}^\beta + \frac{1}{1-n} Q_{\alpha\beta}^\beta Q_{\gamma\delta}^\delta], \quad (18)$$

the Einstein field equations can be written in the DW Hamiltonian form (cf. Eq. (1))

$$\partial_\alpha h^{\beta\gamma} = \partial H / \partial Q_{\beta\gamma}^\alpha, \quad \partial_\alpha Q_{\beta\gamma}^\alpha = -\partial H / \partial h^{\beta\gamma}. \quad (19)$$

The first of Eqs. (19) is equivalent to the well-known expression of the Christoffel symbols in terms of the metric. The second one yields the vacuum Einstein equations $R_{\alpha\beta}(\Gamma) = 0$ for the Christoffel symbols. Note that variables $Q_{\beta\gamma}^\alpha$ play the role of polymomenta associated with the space-time derivatives of the metric density $h^{\alpha\beta}$.

Now, the already familiar canonical brackets for forms constructed from polymomenta $Q_{\beta\gamma}^\alpha$ and field variables $h^{\mu\nu}$ can be written down (cf. Eqs. (2)) and quantized according to Eq. (3). This results in the following expression for the operator $\hat{Q}_{\beta\gamma}^\alpha$:

$$\hat{Q}_{\beta\gamma}^\alpha = -i\hbar\kappa\sqrt{|g|}\gamma^\alpha\frac{\partial}{\partial h^{\beta\gamma}}. \quad (20)$$

Substituting (20) to the classical expression (18) we obtain the following operator of the DW Hamiltonian density^d

$$\hat{H} = -8\pi G\hbar^2\kappa^2\frac{n-2}{n-1}\left\{\sqrt{|g|}h^{\alpha\gamma}h^{\beta\delta}\frac{\partial}{\partial h^{\alpha\beta}}\frac{\partial}{\partial h^{\gamma\delta}}\right\}_{ord}, \quad (21)$$

where the notation $\{\dots\}_{ord}$ refers to the ordering ambiguity of the expression inside the curly brackets. In fact, the form of \hat{H} as written down in (21) corresponds to the arbitrarily chosen “standard” ordering when all differential operators appear on the right hand side. We hope that by requiring the Hermiticity of \hat{H} and applying the correspondence principle we can chose a proper ordering of operators in (21) and obtain a more precise expression for the DW Hamiltonian operator of gravity (work in progress).

Further, if the wave function in (16) is assumed to be a spinor, the operator of the covariant derivative $\hat{\nabla}_\mu$ is $\hat{\nabla}_\mu := \partial_\mu + \hat{\theta}_\mu$, where $\hat{\theta}_\mu$ denotes the operator corresponding to the spinor connection. Classically the spinor connection is given by Eqs. (13-14). From (14) it is clear that the metric formulation does not allow us to construct the operator $\hat{\theta}_\mu$, because it includes the operator corresponding to the first jet components (=derivatives) of tetrads $\partial_\mu e_\nu^A$. This operator can be derived only from the quantization of polymomentum variables associated with the derivatives of tetrads. We thus arrive at the problem of a DW-like formulation of General Relativity in tetrad variables (work in progress). This formulation will, of course, lead to another expression for the DW Hamiltonian density in terms of tetrads and corresponding polymomenta and, consequently, to a different from (21) expression for the DW Hamiltonian operator. The consistency with the Ehrenfest principle can be checked only if the operator corresponding to the spinor connection is found.

5 Discussion

In this communication we have pointed out the approach to the quantization of fields and general relativity which emanates from the manifestly covariant

^dRecall that in n dimensions the Newton gravitational constant $G \sim l_{Pl}^{n-2}/\hbar$.

generalization of the Hamiltonian formalism to field theory known as the De Donder-Weyl theory. Within the latter fields are treated as multi-parameter De Donder-Weyl Hamiltonian systems described by Eqs. (1), with space-time variables entering as parameters similar to the single time parameter in the usual Hamiltonian formalism. The corresponding quantization involves the space-time Clifford algebra as a higher-dimensional generalization of the algebra of complex numbers used in quantum mechanics; in doing so the standard quantum mechanics is derived as a special case corresponding to the one-dimensional “field theory”. Notice that some of our results do not necessarily require γ^μ -s in the above formulas to be the Dirac matrices. However, the question whether or not other hypercomplex systems appearing in various first order relativistic wave equations¹⁶ could be suitable for the present framework has not been studied by us.

The description of quantized fields is achieved here in terms of partial differential equations and functions over a finite dimensional covariant analogue of the configuration space – the space of field and space-time variables. This description appears to be very different as compared with, say, the Schrödinger functional representation in quantum field theory which employs an infinite dimensional configuration space, functionals, and functional derivative equations. Still, a qualitative argument based on the physical meaning of our wave function and of the Schrödinger wave functional suggests that a connection between both can exist (see⁷ for preliminary discussion): while our wave function $\Psi(y, \mathbf{x}, t)$ is the probability amplitude that a field has the value y in the space point \mathbf{x} in the moment of time t , the Schrödinger functional $\Psi([y(\mathbf{x})], t)$ is the probability amplitude that a field has the configuration $y(\mathbf{x})$ in the moment of time t . Hence, the Schrödinger functional could in principle be related to a certain composition of single amplitudes given by our wave function.

There are several open questions regarding the generalized Schrödinger equations (4), (11), (16). Particularly, it is not quite clear yet whether or not the wave function Ψ should be a (universal) Dirac spinor for any field (of any spin) under quantization. If we accept the idea of a universal spinor wave function in (4) the question naturally arises as to how this can be reconciled with the spin content of different fields we are to quantize. This problem is similar to the early attempts to construct matter from a universal spinor field, such as the Heisenberg nonlinear spinor theory and De Broglie’s “neutrino theory of light”, which are nowadays abandoned.

On the one hand, a possible way out may be that equation (4) (and (11), (16)) should be understood rather as one of the system of $2s$ equations of the Bargmann-Wigner type¹⁶ for a *multispinor* wave function with $2s$ (antisymmetric) indices, which describes particles of spin $s \geq 1/2$ (the case of a

scalar field would have to be given a separate treatment then). However, it is not clear to us how this point of view can be extended to systems like QED which involve interacting fields of different spins.

On the other hand, if Ψ is taken to be a general element of the Clifford algebra its component content may probably suffice to incorporate fields of different spins (including spinors if viewed as minimal left ideals¹⁰). In this light, our earlier assumption⁷ that Ψ is a hypercomplex valued function (or a non-homogeneous differential form¹⁴), which we have abandoned in⁸ because of difficulties (which probably can be surmounted) with the probabilistic interpretation and the Ehrenfest principle, could still be worthy of further analysis. Moreover, when passing on to curved space-time the use of (multi)spinor wave functions restricts the applicability of the present framework to space-times which admit spinor structure. Such restriction seems to be rather artificial, especially if we think about a subsequent application to quantization of General Relativity. By requiring that no restriction to spin space-time manifolds is allowed we arrive at yet another alternative by using the Dirac-Kähler equation^{10,15} rather than the Dirac spinor equation.

Let us recall that in Minkowski space-time the Dirac-Kähler equation leads to the same predictions as the Dirac equation. However, in curved space-time it exhibits different behavior which is incompatible with the Pauli exclusion principle¹⁰. This feature, however, cannot be treated as a drawback if the Dirac-Kähler equation is used not to describe the electron with spin but as a more fundamental equation.

The deep geometric content of the Dirac-Kähler equation reconciles remarkably with the geometric spirit of General Relativity. On this ground the Dirac-Kähler version of (16) can be put forward as a better (although still hypothetical) candidate for description of quantum General Relativity

$$i\hbar\kappa\sqrt{|g|}\widehat{\mathcal{D}}_\Gamma\Psi=\widehat{H}\Psi. \quad (22)$$

Here $\mathcal{D}_\Gamma:=d_\Gamma-\delta_\Gamma$ is the Dirac-Kähler (Hodge-de Rham) covariant differential operator related to the metric connection Γ , with the hat over it in (22) denoting that the connection coefficients $\Gamma^\alpha_{\beta\gamma}$ are differential operators themselves (cf. Eq. (20)). The wave function in (22) is a non-homogeneous form: $\Psi=\sum_{p=0}^n\frac{1}{p!}\psi_{\mu_1\dots\mu_p}dx^{\mu_1}\wedge\dots\wedge dx^{\mu_p}$, and the operators have to be represented using the elements of the Atiyah-Kähler algebra (e.g. $\partial_\mu\lrcorner$ and $dx^\mu\wedge$) rather than Dirac matrices. In this formulation we already do not need to employ the tetrad formulation of gravity. All the quantities can in principle be determined from quantization of metric gravity in the DW formulation. Potential difficulties which may emerge again from the operator ordering ambiguity probably can be overcome with the help of the correspondence principle.

Further work will hopefully tell us which of the alternatives outlined above is more consistent; what is the physical content of the presented formalism, and how it can be linked to the contemporary quantum field theory.

When the paper was being written the preprint by M. Navarro¹⁷ has appeared in which he discusses an approach to a “finite dimensional” quantization of fields similar to ours in Sect. 2.

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